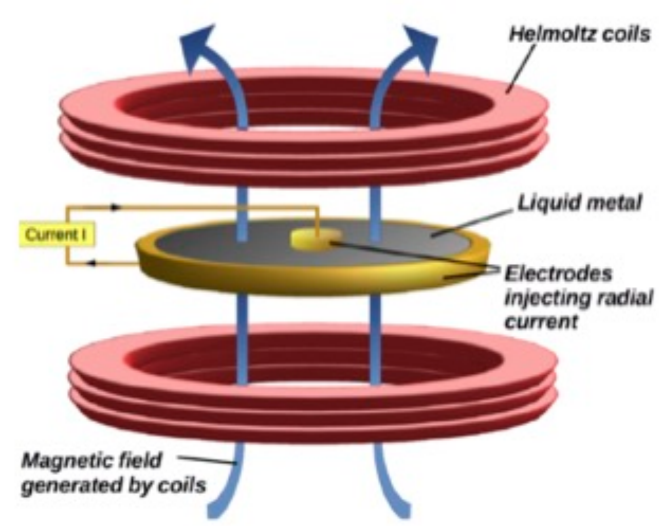
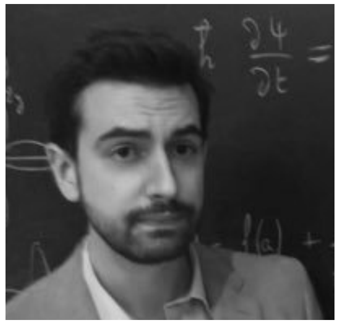
Angular momentum transport by astrophysical turbulence

Christophe Gissinger

Ecole Normale Supérieure, Paris





Michael Pereira
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Marlone Vernet
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ENS
ÉCOLE NORMALE SUPÉRIEURE

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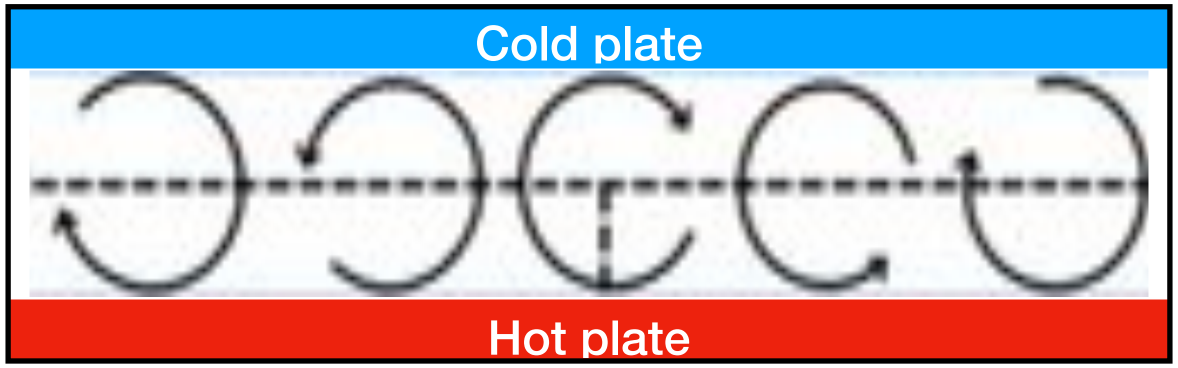
TRANSPORT BY TURBULENCE

An exemple: transport of heat in Rayleigh-Benard



- Heat : $H = \rho c_p T$

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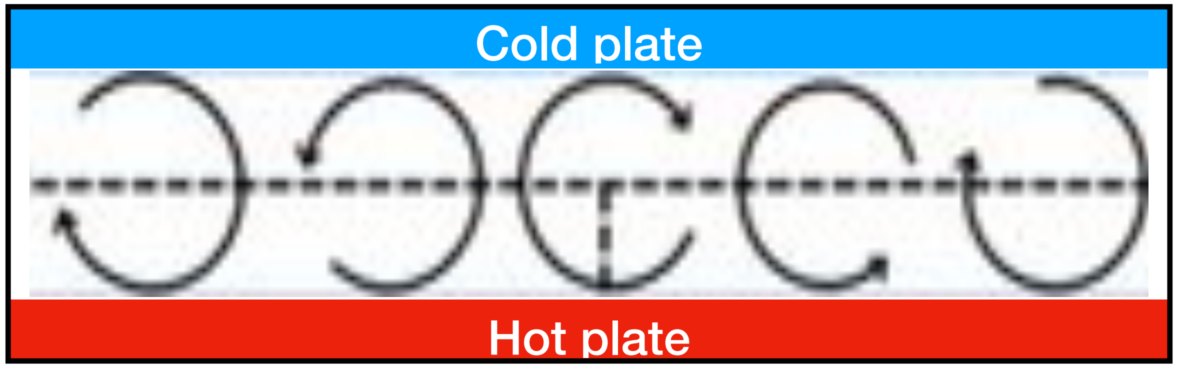

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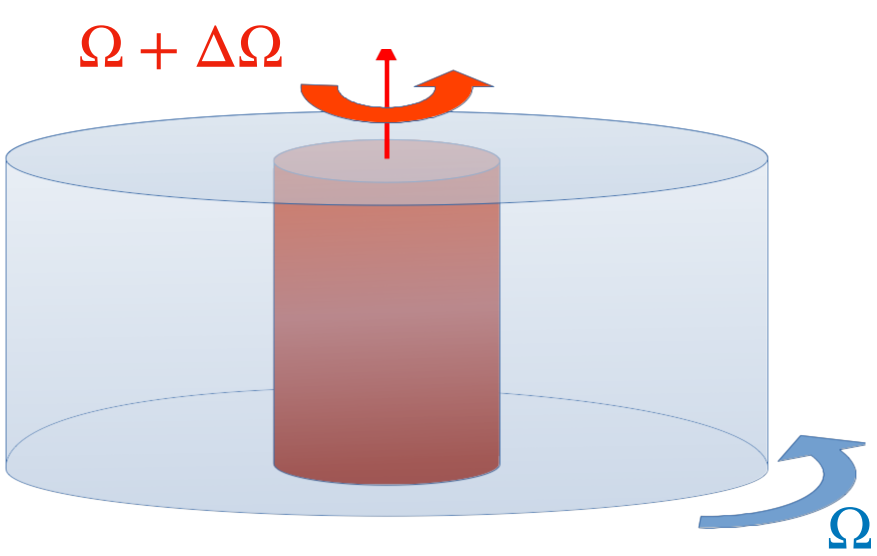
Transport of angular momentum in rotational flows



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TRANSPORT BY TURBULENCE

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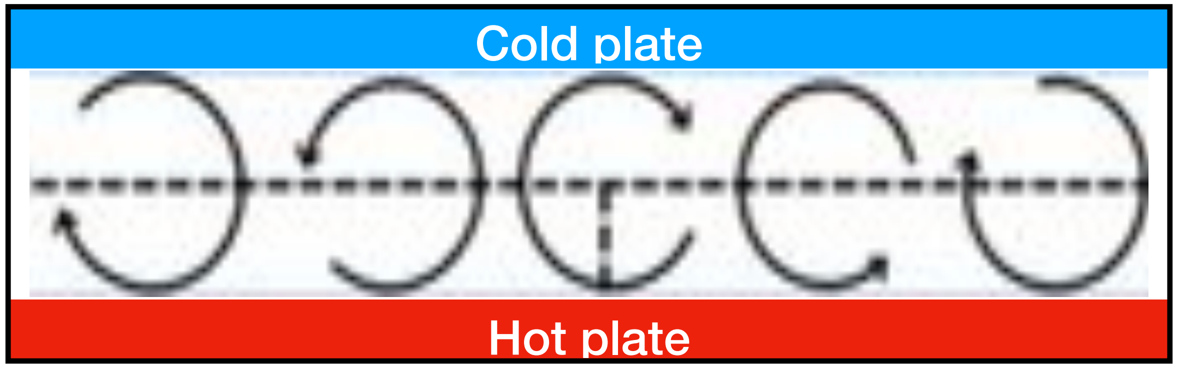

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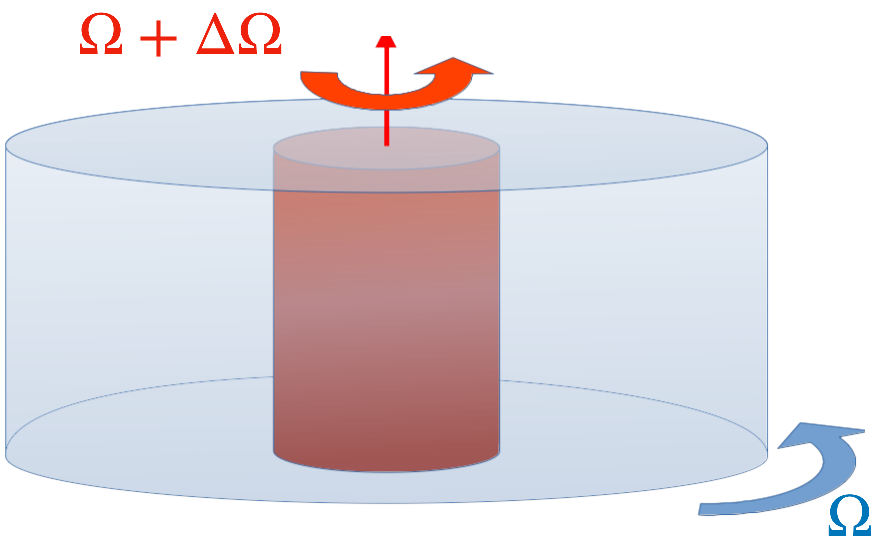

- Angular momentum (AM) : $\Gamma = \Omega r^2$

→ **Is it possible to understand and predict the angular momentum flux** $J_\Omega = f(\Delta\Omega, \nu)$ **?**

→ **Role of the boundary conditions on the turbulent transport ?**

ASTROPHYSICAL MOTIVATION

Astrophysical turbulence is ubiquitous, but tricky to explain …

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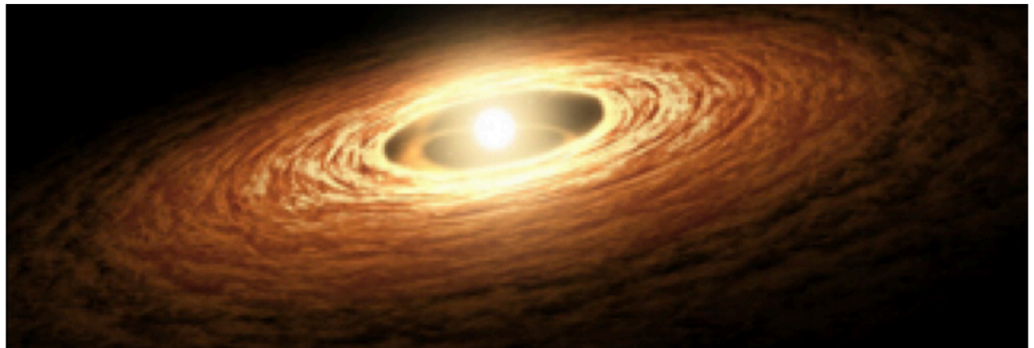
Artist's illustration of EX Lup in outburst (NASA/JPL-Caltech/T. Pyle)

- Huge accretion rate ⇔ Outward transport of angular momentum
- Weak turbulence

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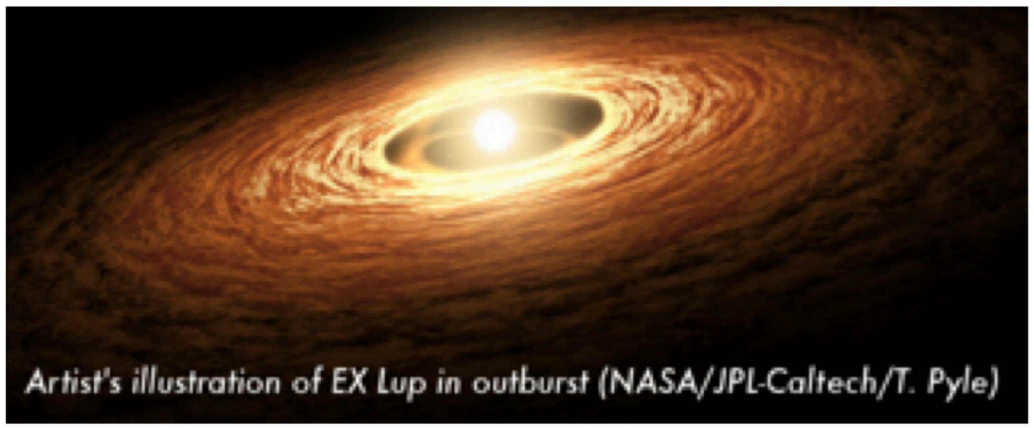
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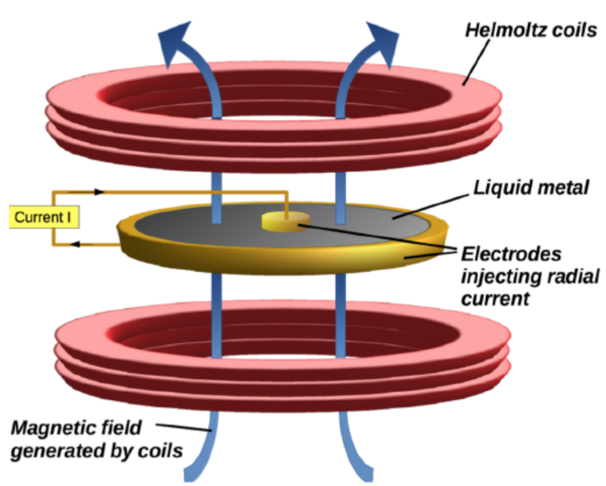
AN ACCRETION DISK IN THE LABORATORY

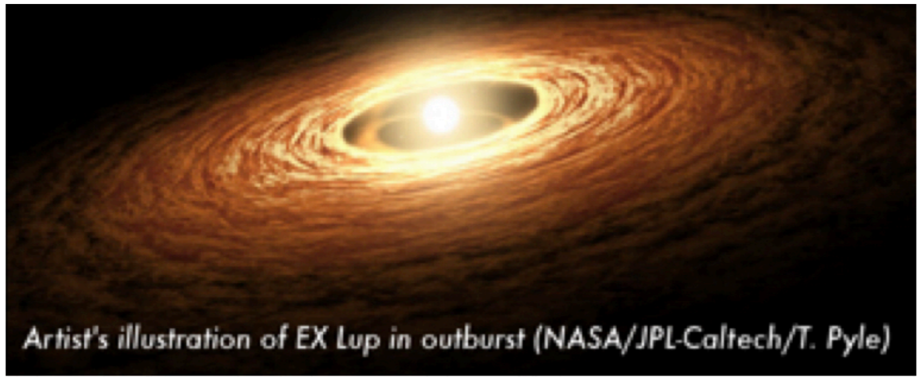


Liquid metal experiment:

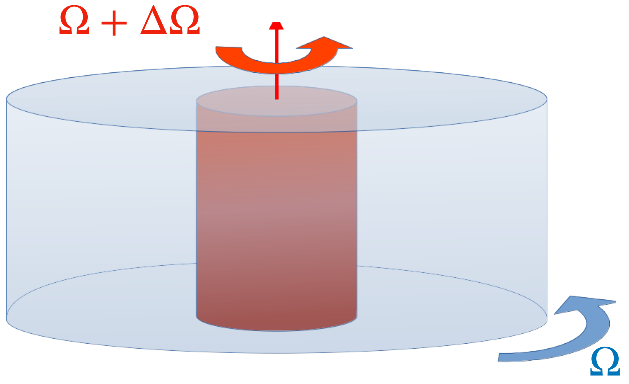
- Not an MRI experiment !
- Prediction for accretion rates
- Ultimate regime for angular momentum transport

LABORATORY MODELS OF ACCRETION DISKS

ACCRETION DISKS



TAYLOR-COUETTE FLOWS



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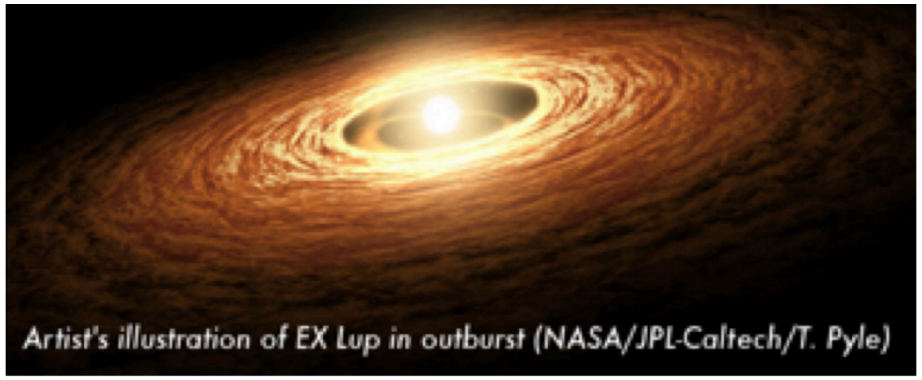

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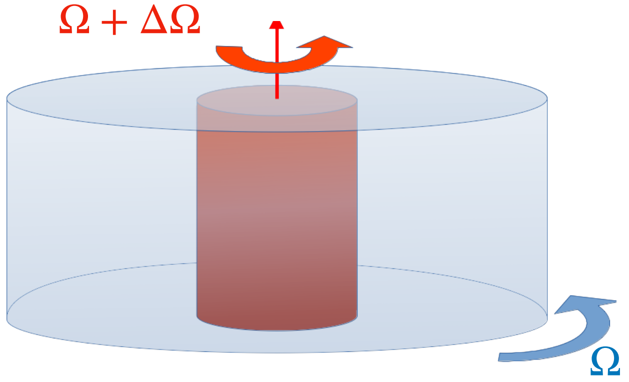

Limitations due to the use of Taylor-Couette flows:

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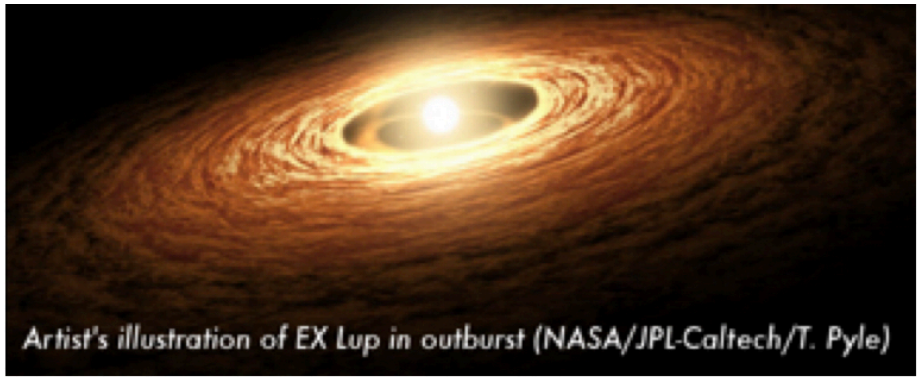

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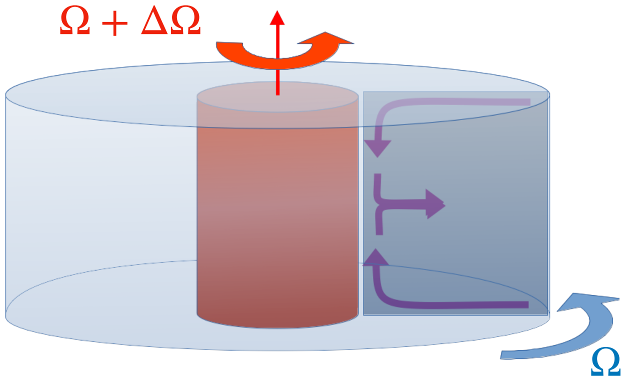

Limitations due to the use of Taylor-Couette flows:

I - Keplerian rotation: $\frac{u_\varphi^2}{r} \sim \frac{GM}{r^2} \Rightarrow u_\varphi = \frac{K}{\sqrt{r}}$

(linearly stable, but weakly turbulent)

I - Couette profile $u_\varphi = Ar + \frac{B}{r}$

(either quasi-Keplerian or fully turbulent)

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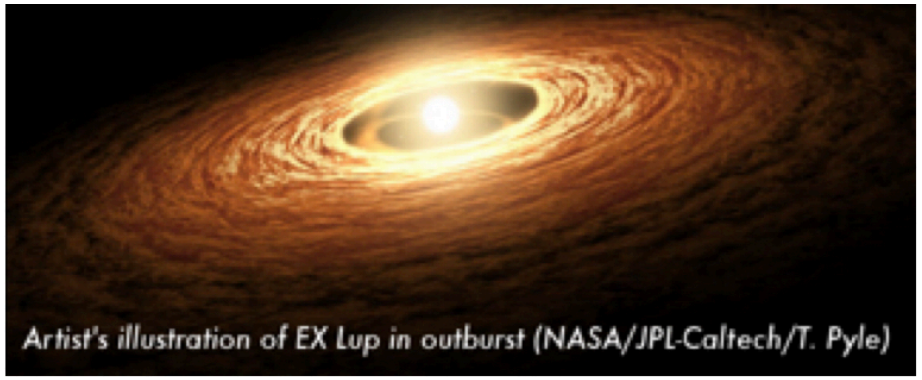

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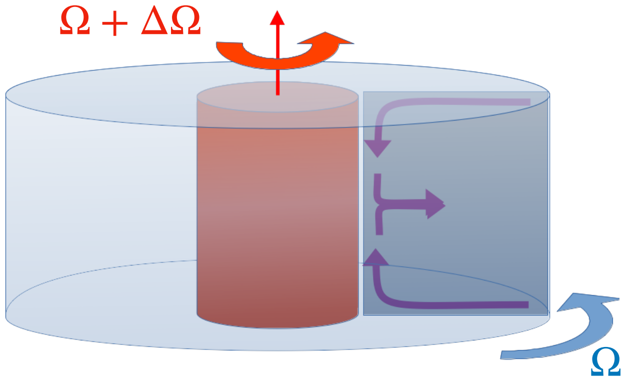

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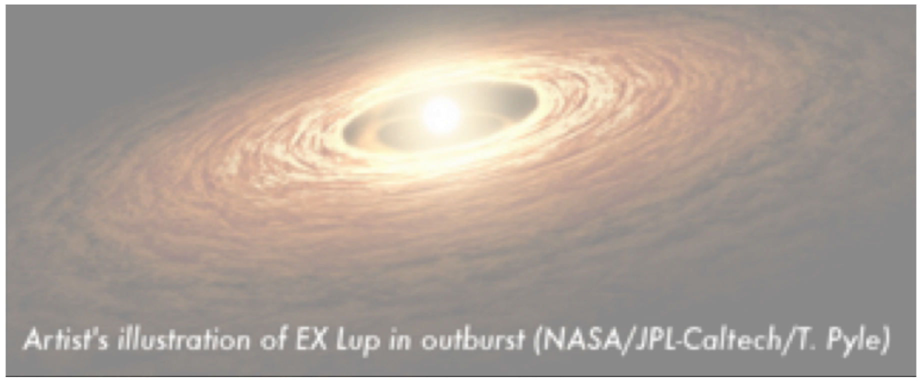

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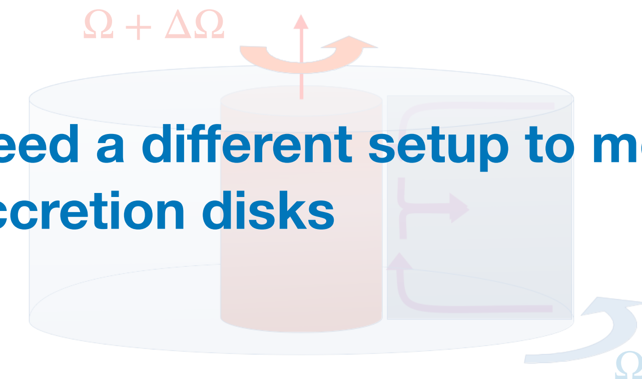

Need a different setup to model accretion disks

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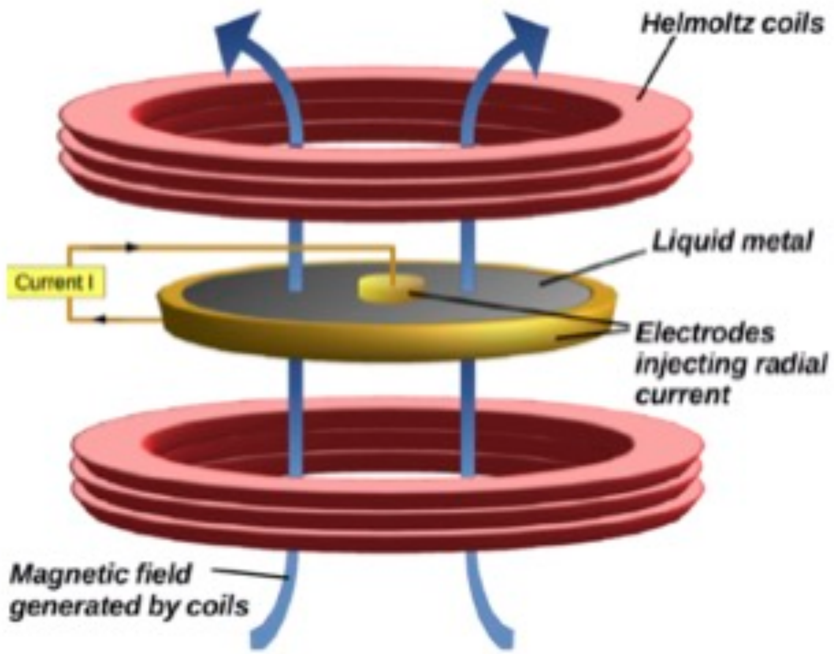

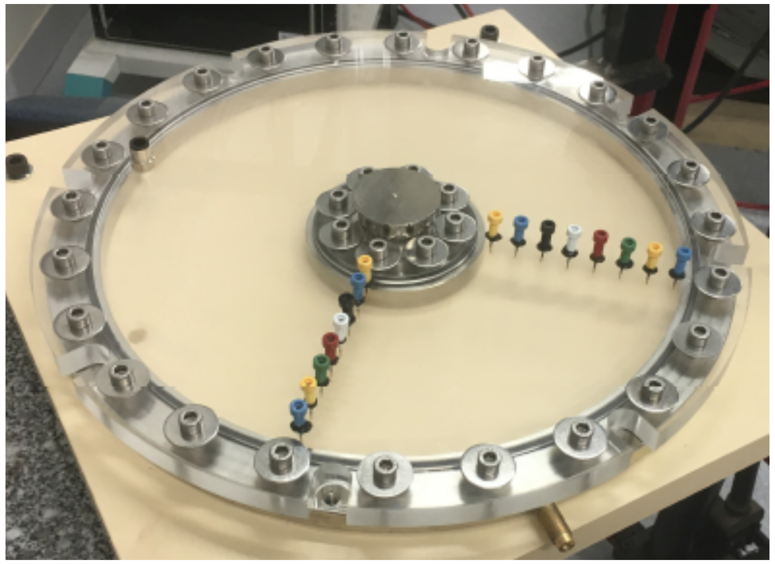

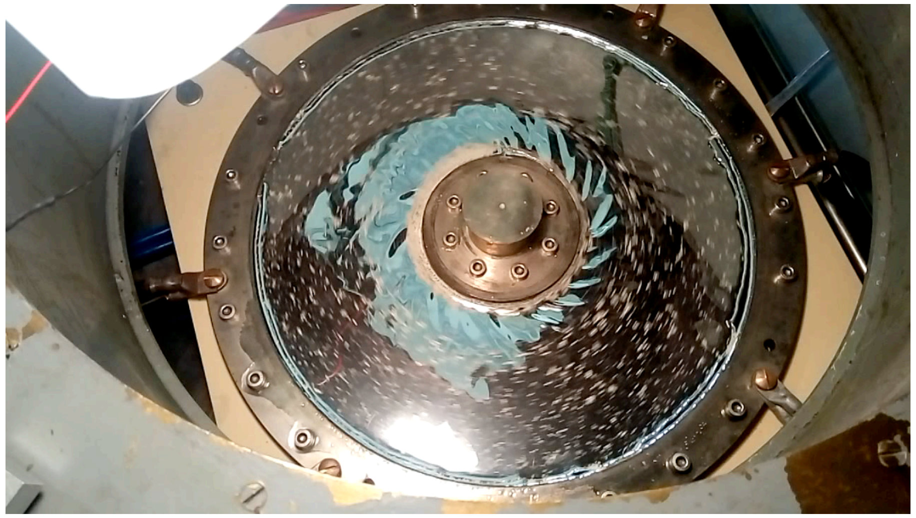

The Kepler experiment

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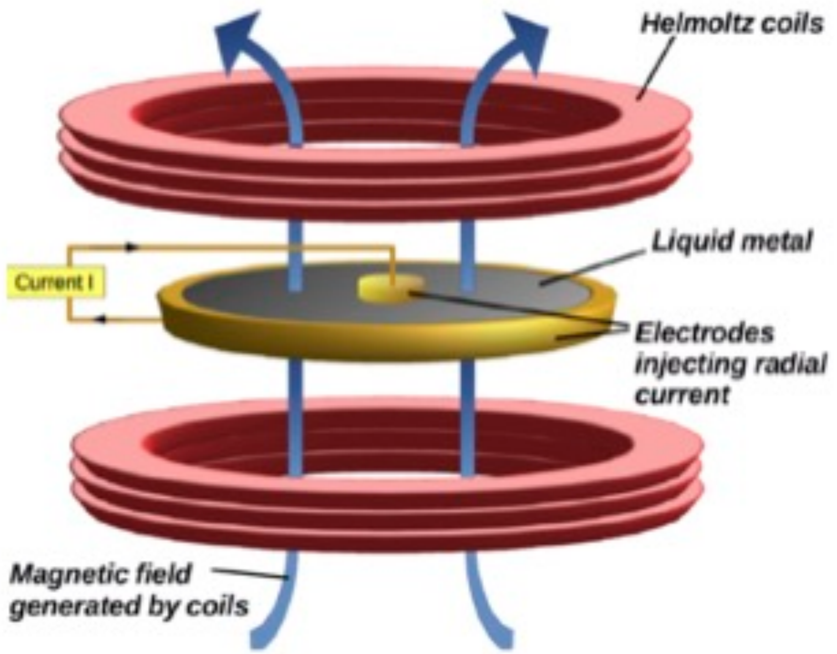

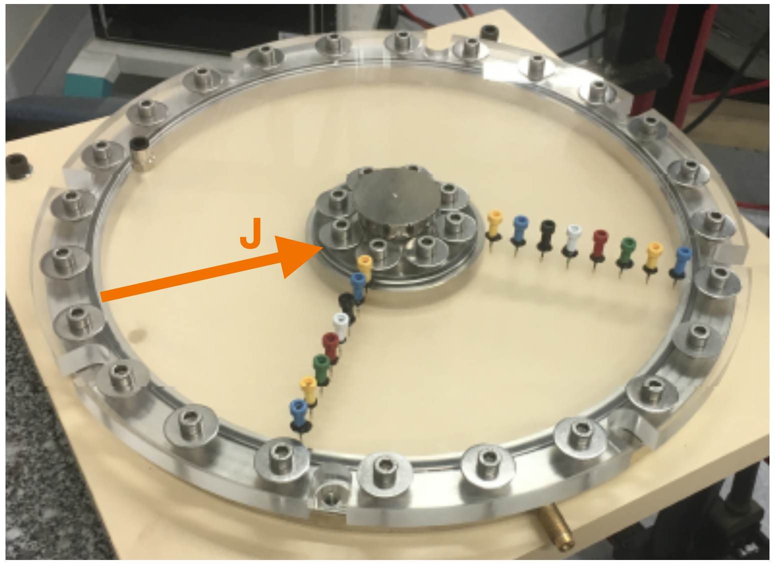

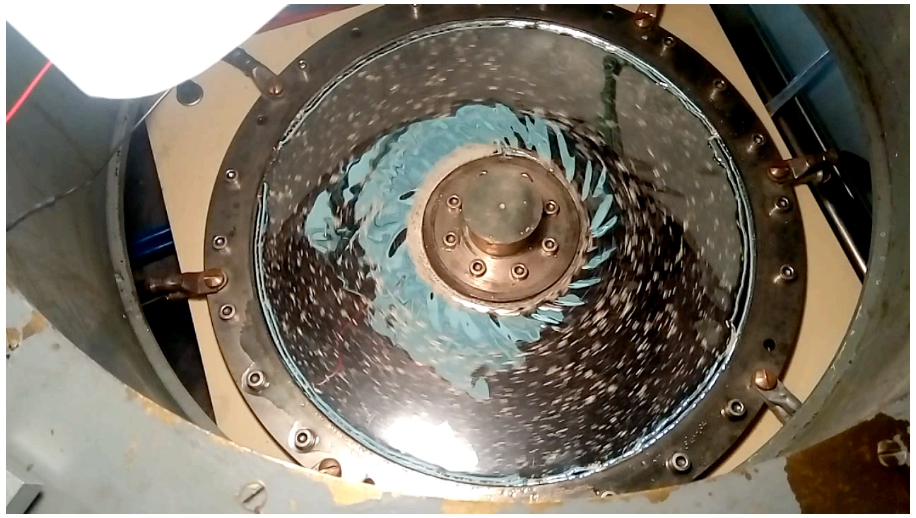

→ **The Lorentz force $\mathbf{j} \times \mathbf{B}$ drives the flow in the azimuthal direction**

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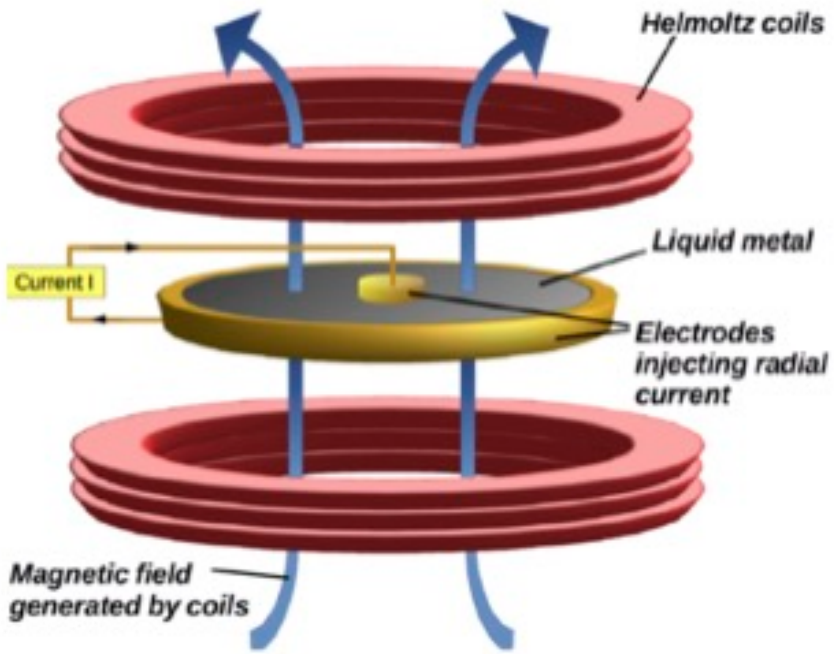

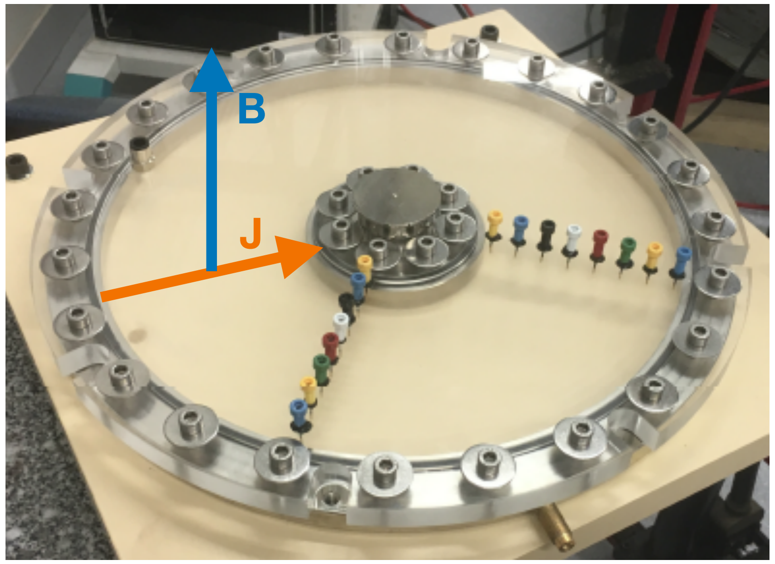

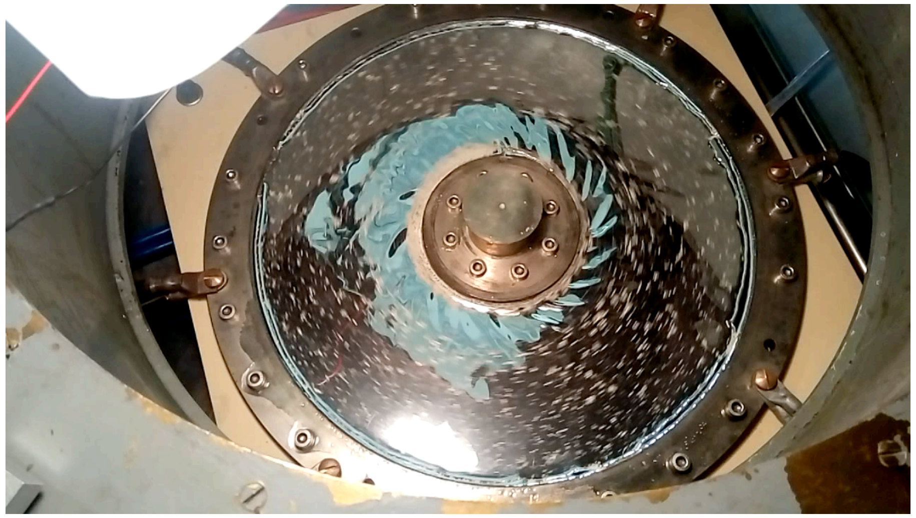

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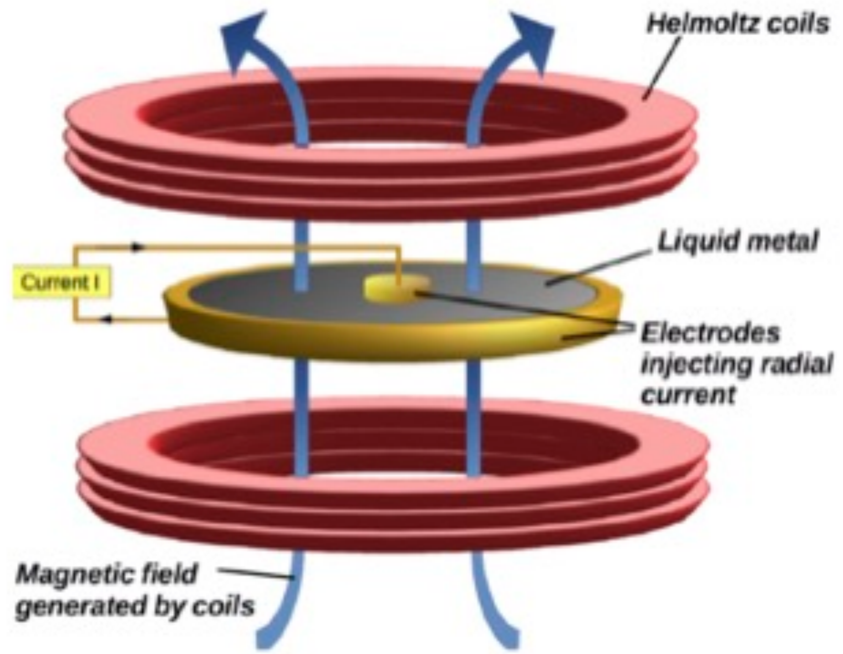

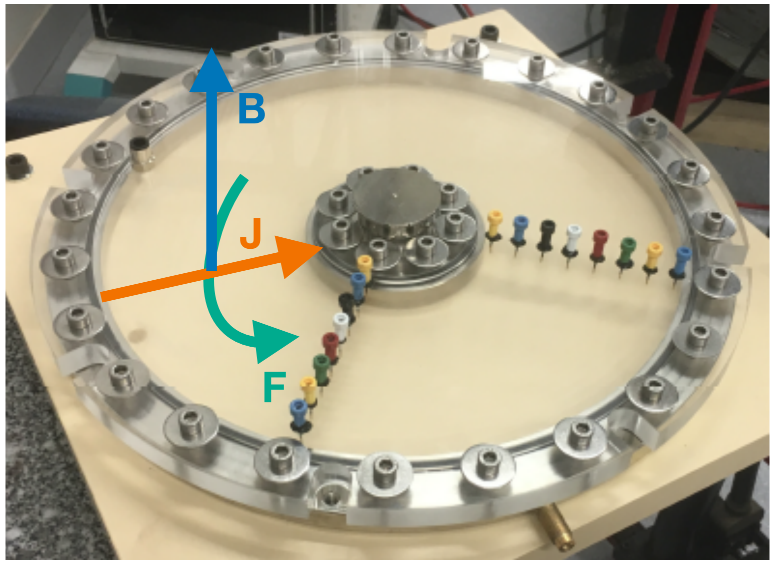

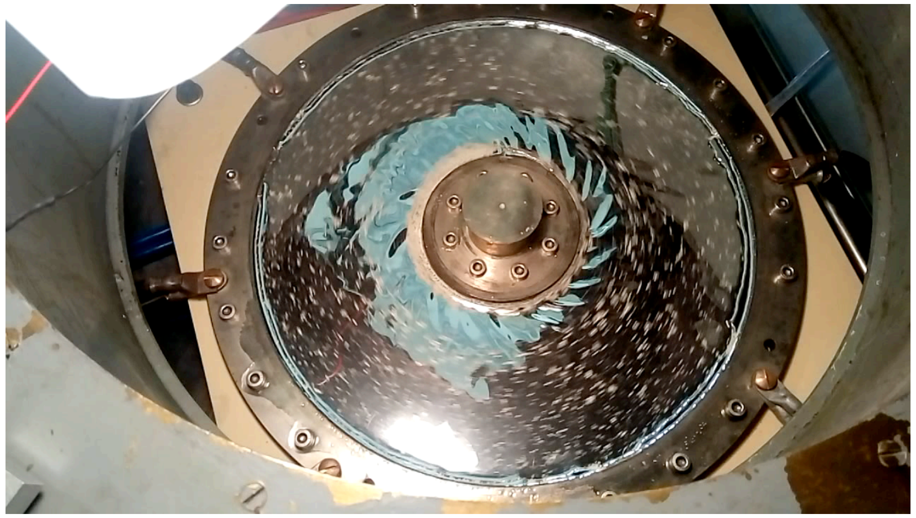

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The Kepler experiment

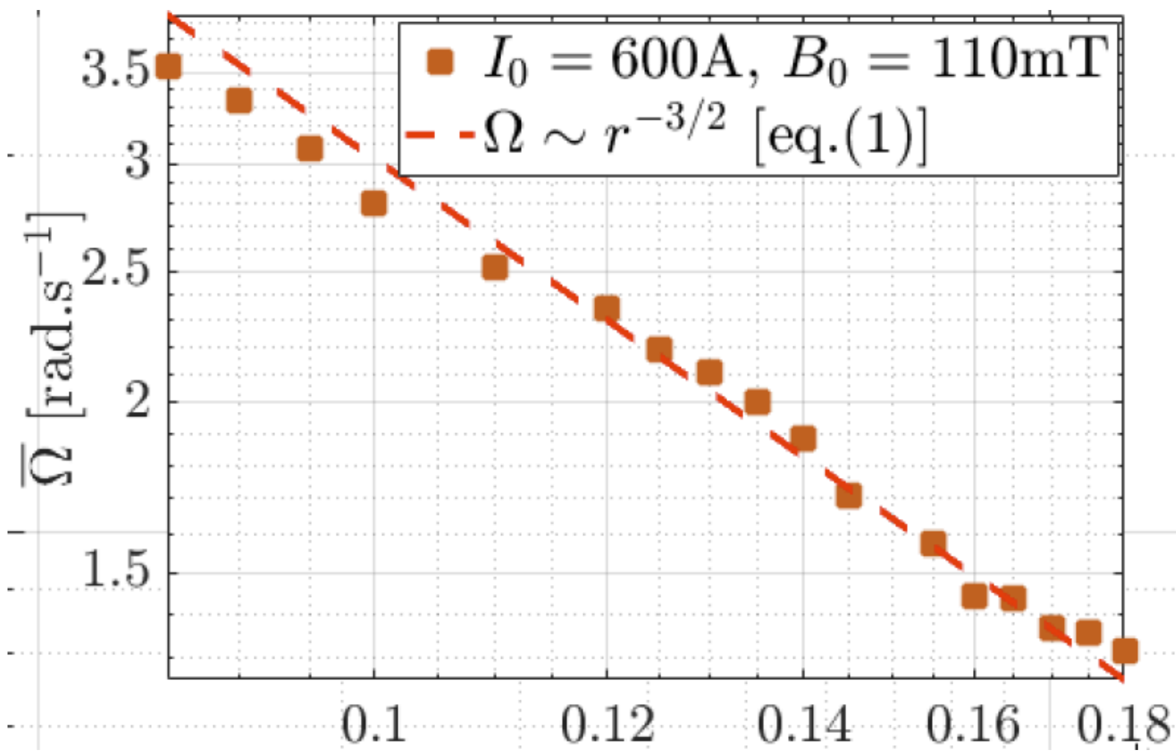
Geometry and control parameters:

- Liquid gallium
- Thin disk, h=1.5cm , D=40 cm
- Radial electrical current I~3000A, Magnetic field B~100mT
- Temperature control

Measurements:

- Radial and azimuthal velocity (*US Doppler velocimetry, potential probes*)
- Pressure fluctuations
- Induced magnetic field (*Hall probes*)

KEPLERIAN TURBULENCE



- Efficient driving of the flow, up to a few m/s
- An **exact** Keplerian rotation rate

(*Vernet et al, JFM. 924, A29 (2021)*)
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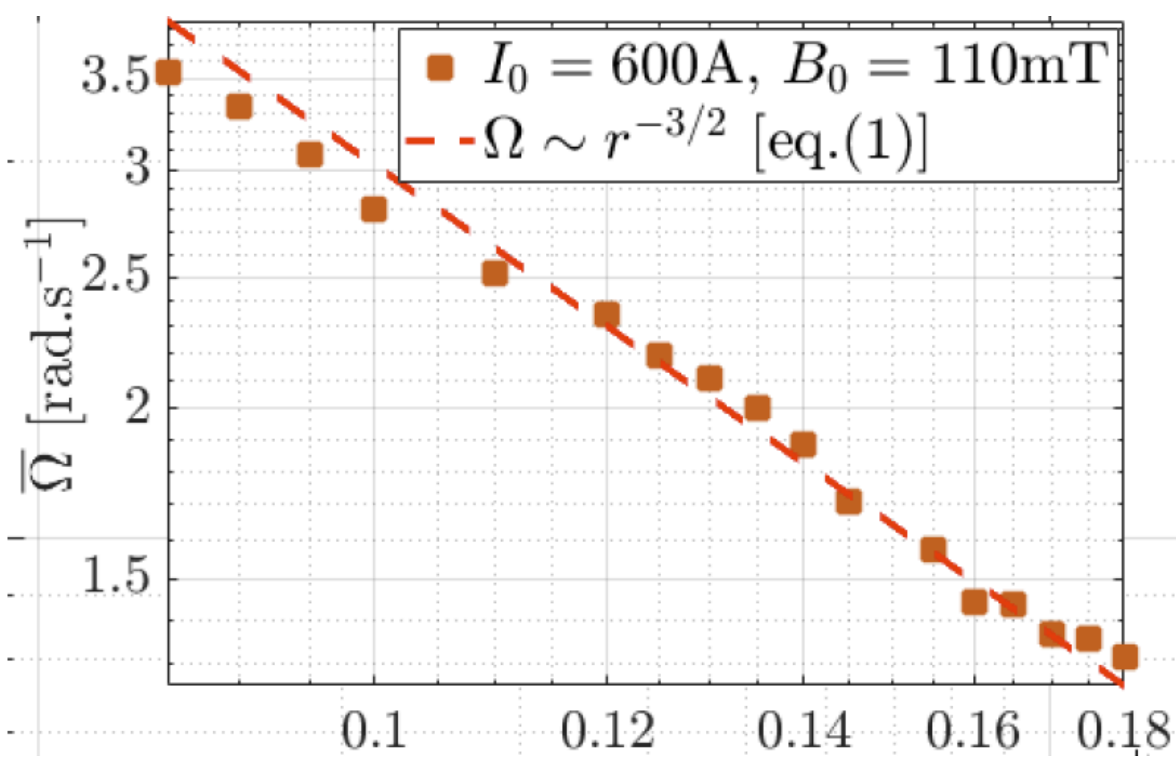

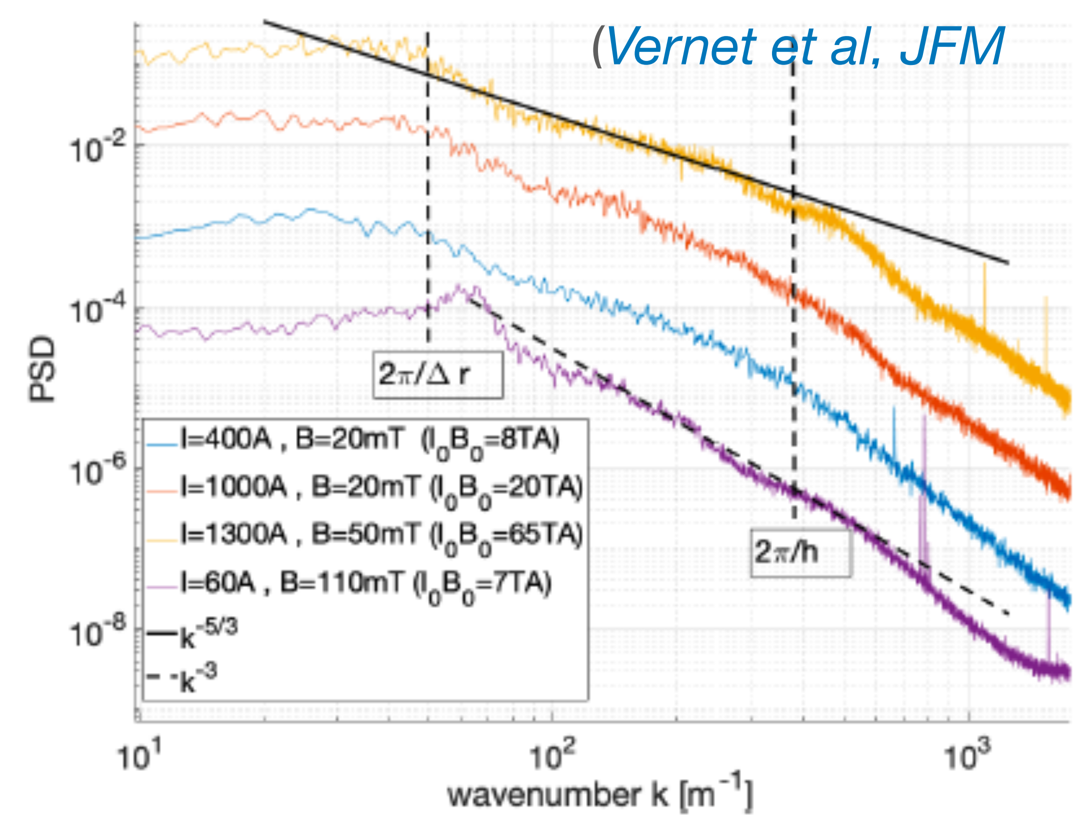

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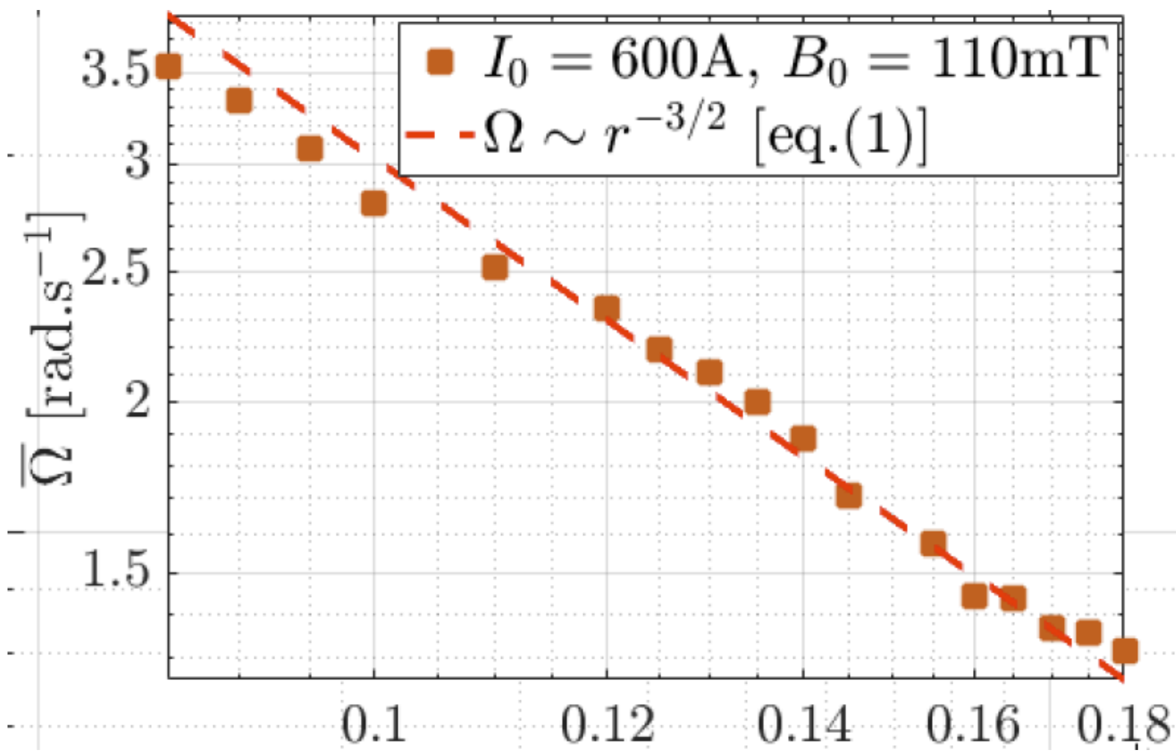

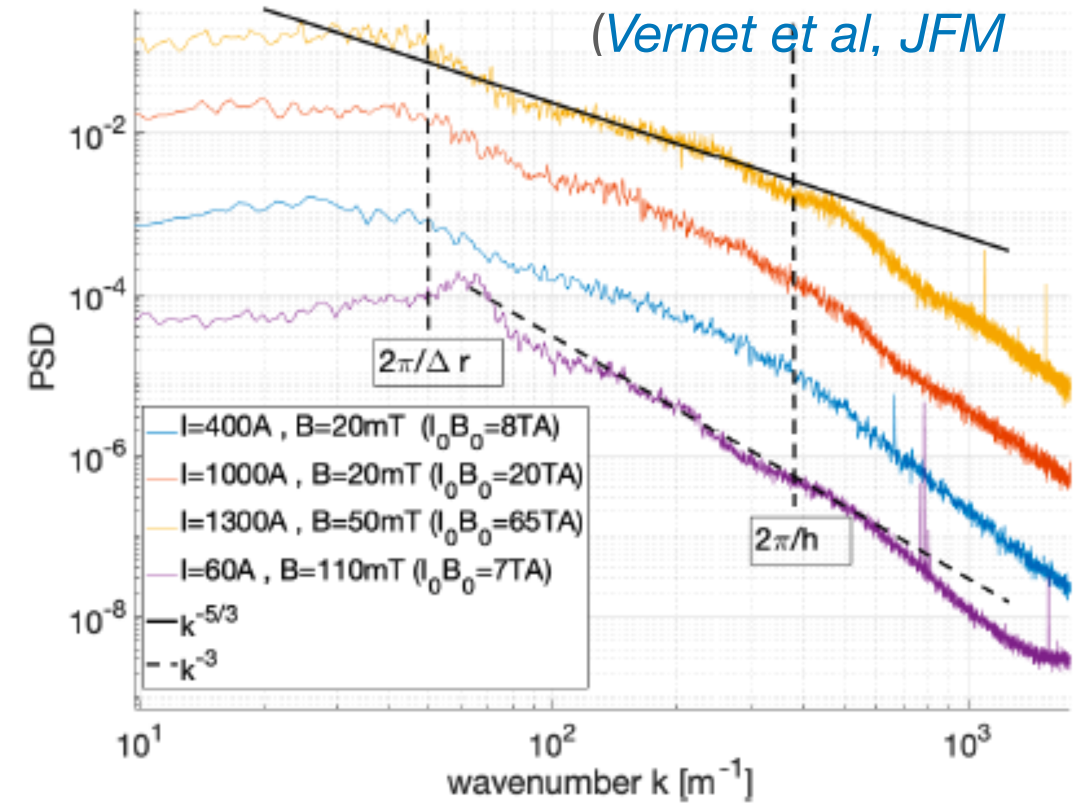

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$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\,.\,\nabla)\mathbf{u}\right) = -\nabla P + \rho\nu\nabla^2\mathbf{u} + \mathbf{j}\times\mathbf{B}$$

$$U_\varphi = \frac{\ln Re}{\kappa}\sqrt{\frac{I_0 B_0}{4\pi\rho r}}$$

→ Unstable boundary layers generate the turbulence, but are not involve in the angular momentum injection

→ First laboratory model of a **thin, turbulent disk in Keplerian rotation and subjected to a magnetic field**

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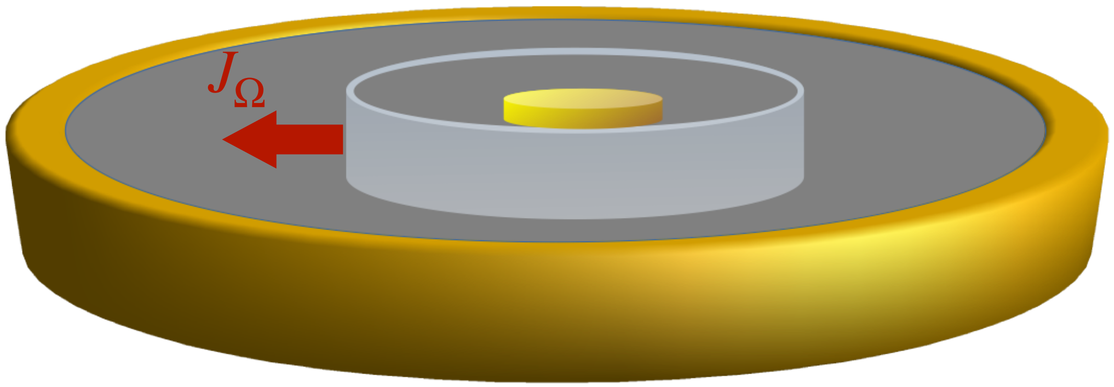
SCALING LAW FOR ANGULAR MOMENTUM TRANSPORT



Angular momentum flux :

$$J_\Omega = r^3 \left(\langle u_r \Omega \rangle - \nu \partial_r \langle \Omega \rangle \right)$$

Efficiency of the turbulent transport:

→ Nusselt number $$Nu = \frac{J_\Omega}{J_\Omega^{lam}} = \frac{J_\Omega}{2\nu r^2 \Omega}$$

Magnitude of the turbulence :

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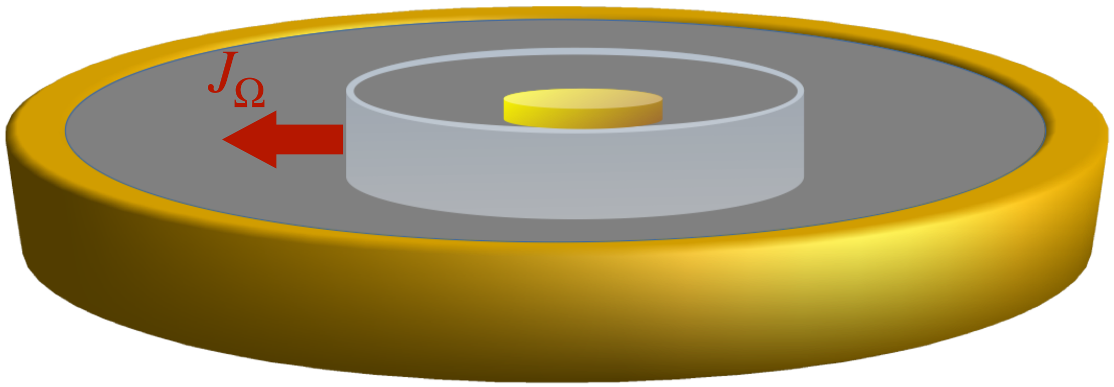

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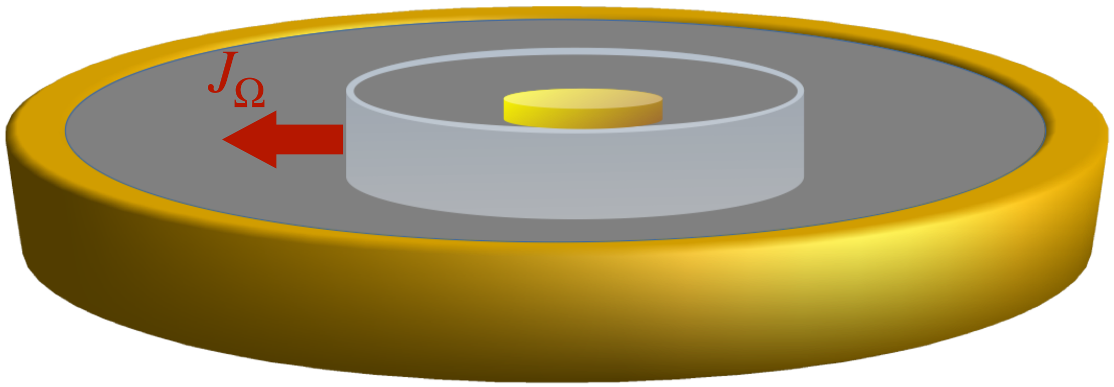

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Kraichnan's ultimate regime (1962):

$$\text{Nu} = \text{Ta}^\beta \Rightarrow J_\Omega = 2\nu r^2 \Omega \left(\frac{\Omega^2 r d^3}{\nu^2} \right)^\beta$$

J_Ω must be independent of ν for sufficiently turbulent flows → $\beta = 1/2$

$$\Rightarrow \mathbf{Nu} = \sqrt{\mathbf{Ta}}$$

BUT :

Previous experiments rather reported $\beta = 0.38$ due to the effect of boundary layers …

(Huisman et al 2012, Gils et al 2011)

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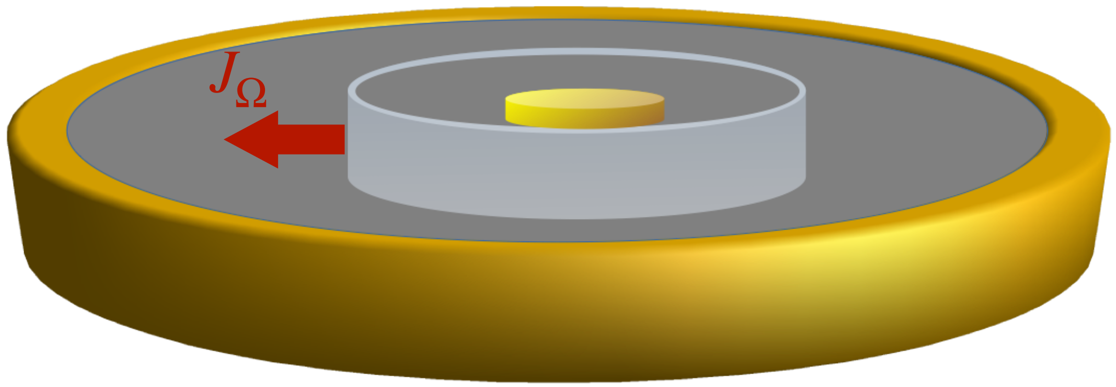

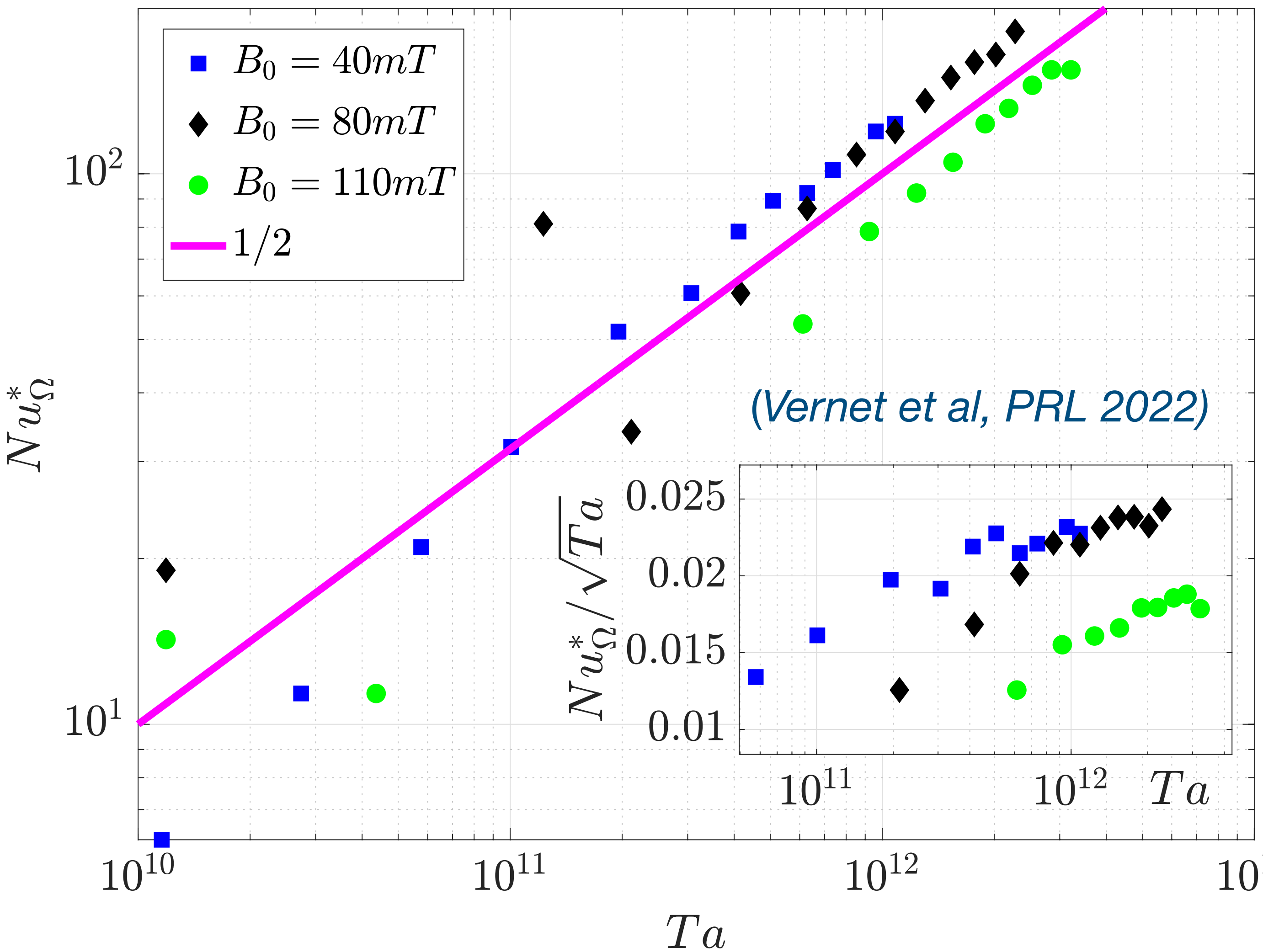

- *Clear-cut scaling law $Nu^*_\Omega \sim \sqrt{Ta}$ on several decades*

*→ This **turbulent** keplerian flow exhibits the Kraichnan ultimate regime*

Kraichnan's ultimate regime (1962):

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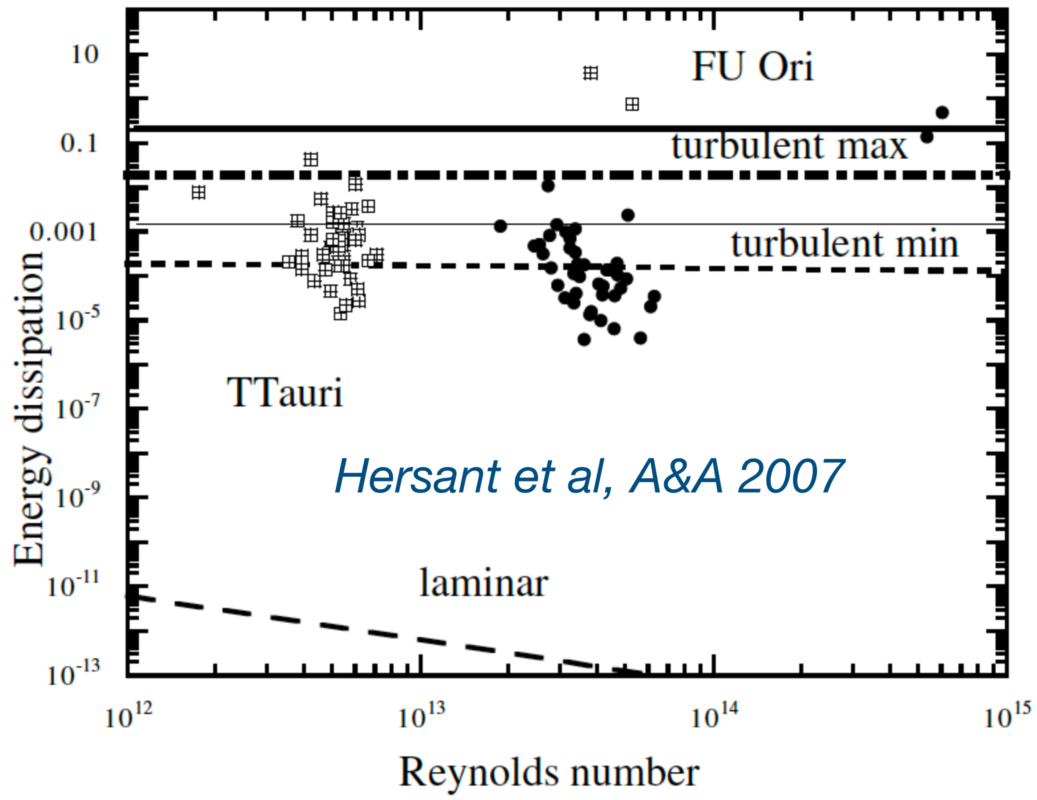
PREDICTION FOR ASTROPHYSICAL ACCRETION DISKS

II - DIMENSIONLESS ENERGY DISSIPATION IN DISKS

$$\mathscr{D} = \frac{\dot{M}}{\dot{M}_0} = \frac{0.8 r^3 \mathscr{L}}{\mathscr{G} M \Sigma \Omega H^4}$$



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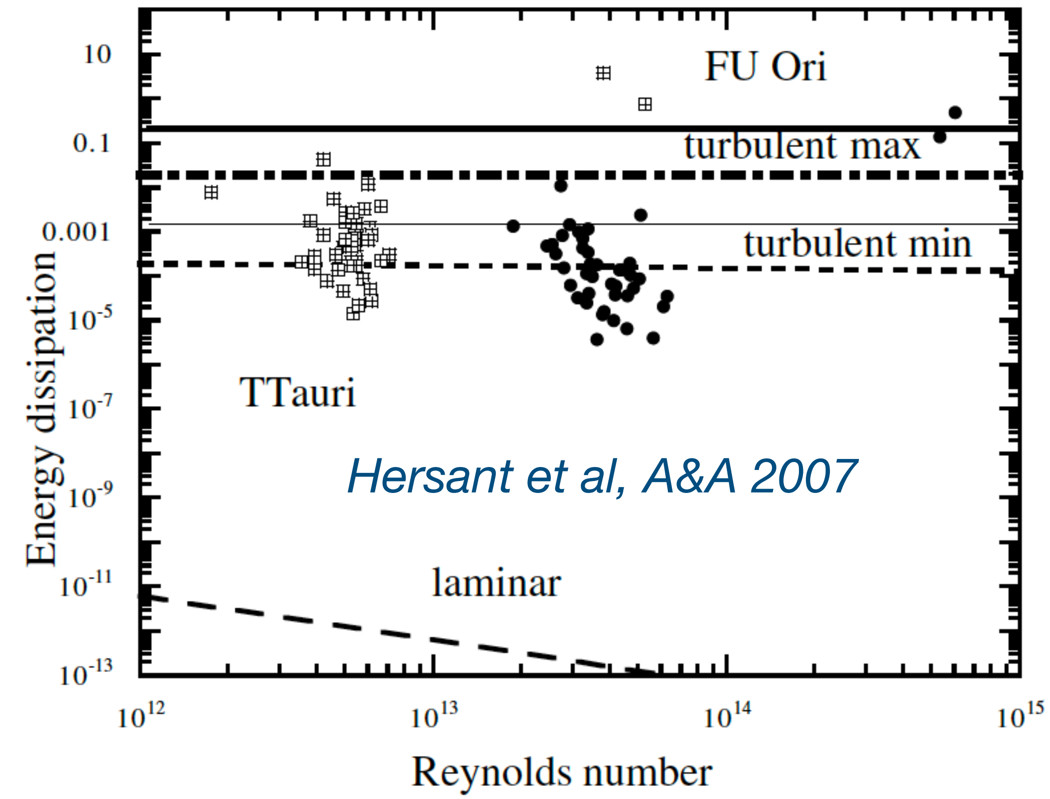

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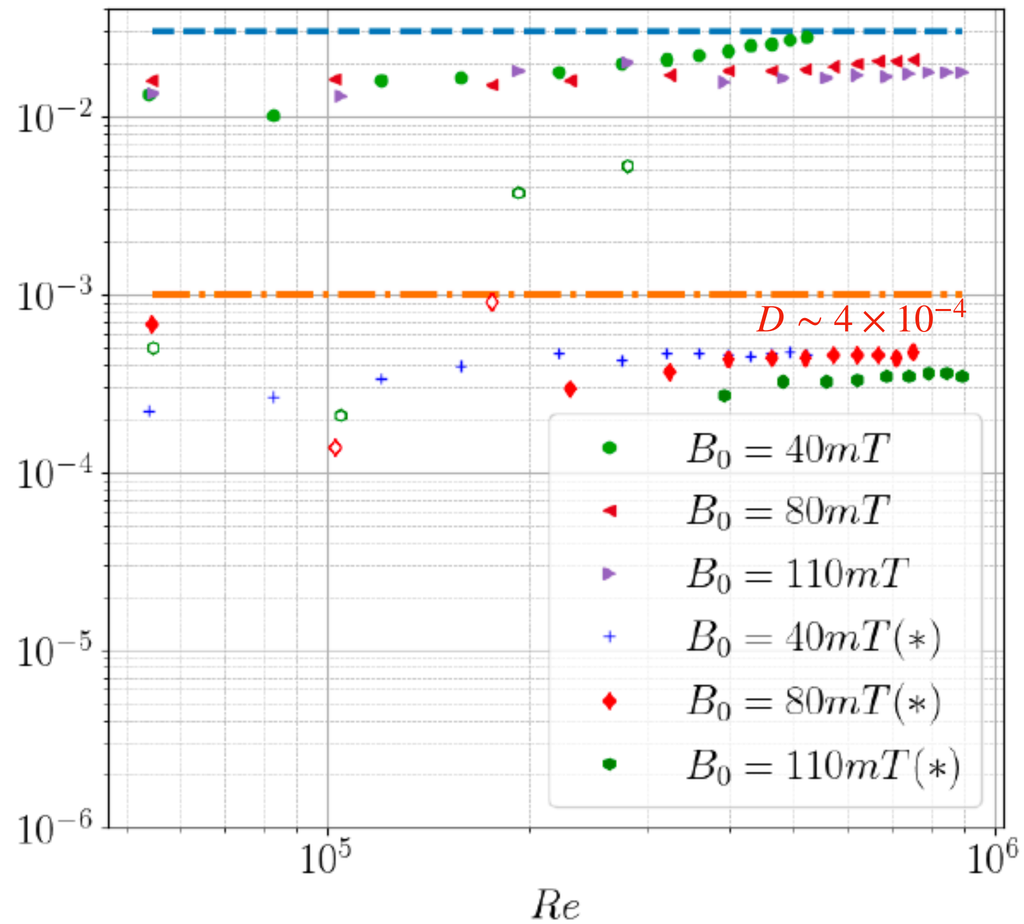

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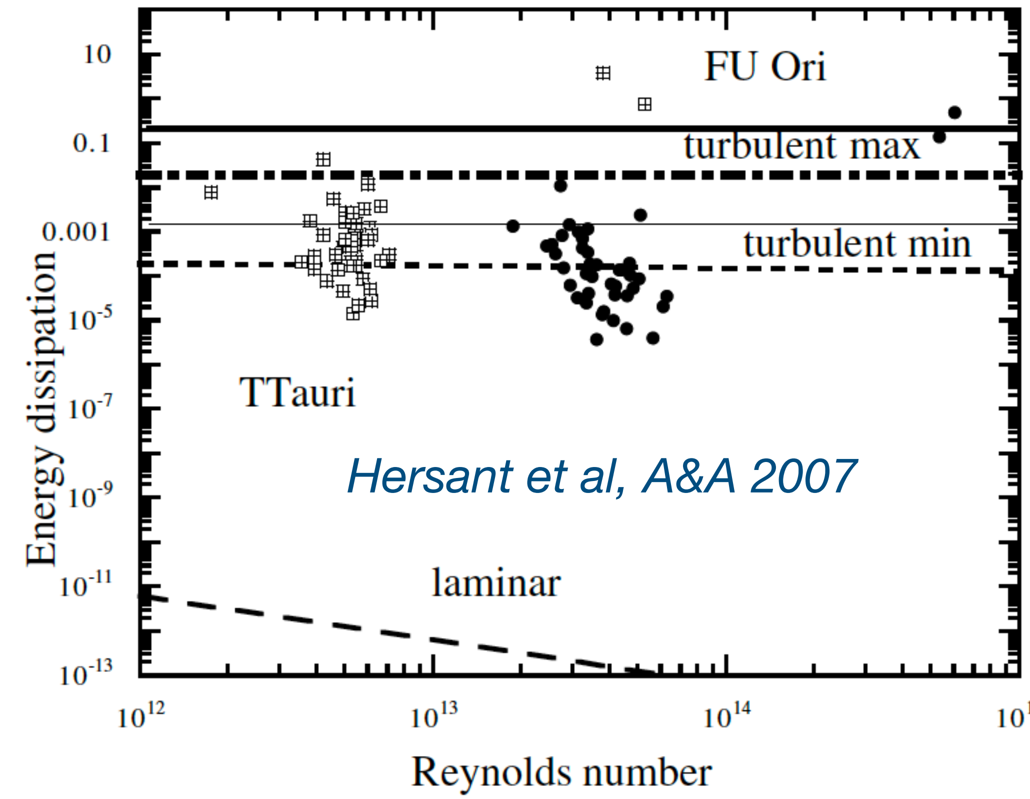

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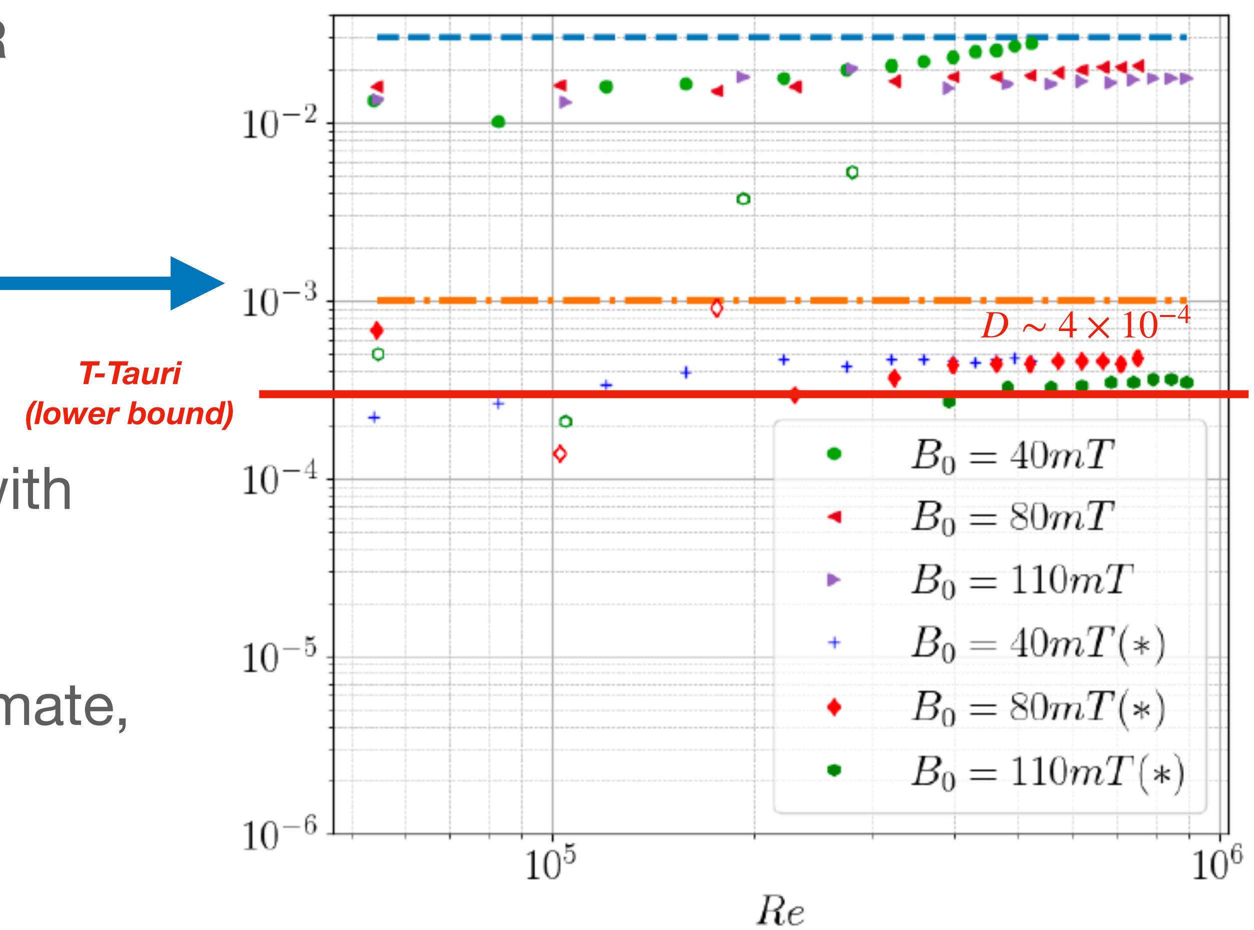

- Predicted accretion rates in very good agreement with observations
- Energy dissipation much smaller than previous estimate, compatible with recent results on *weak turbulence* (*Flaherty et al, ApJ, 2015* : $v_{turb} \sim 0.05 c_S$)

CONCLUSION

Laboratory model of an accretion disk:

- interesting alternative to Taylor-Couette flows (not MRI !)
- Ultimate viscosity-free transport of angular momentum
- Prediction for accretion disks

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A universal picture for turbulent transport, independent of the origin of turbulence ?

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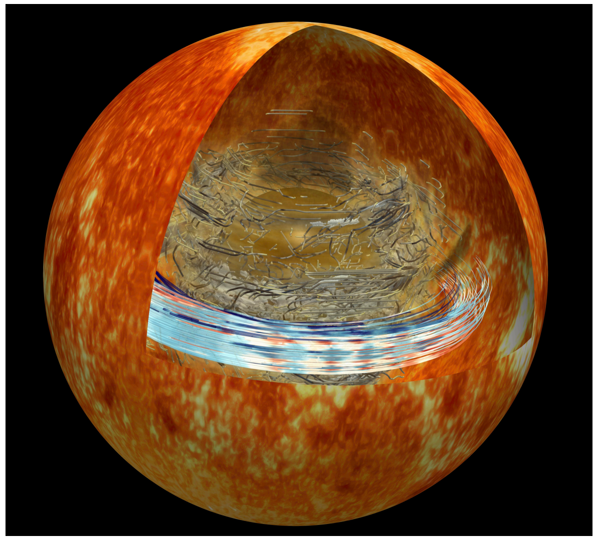
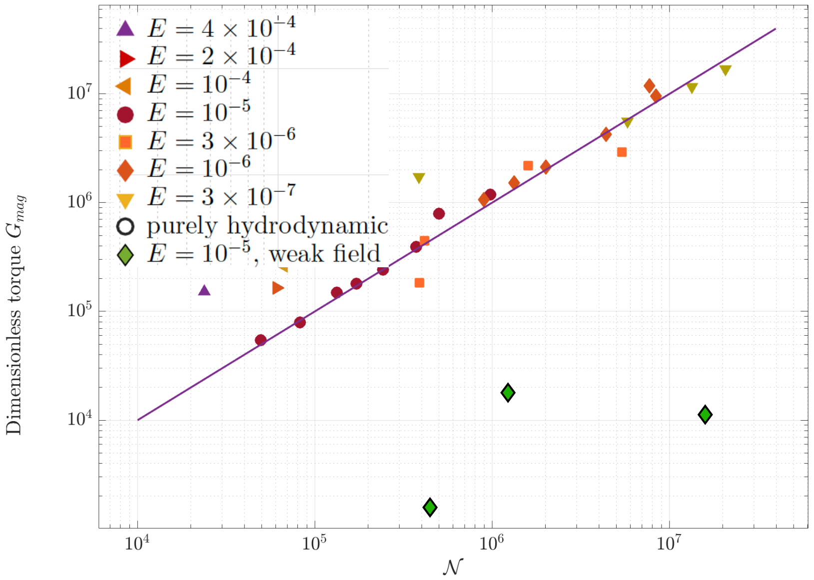
- interesting alternative to Taylor-Couette flows (not MRI !)
- Ultimate viscosity-free transport of angular momentum
- Prediction for accretion disks

A universal picture for turbulent transport, independent of the origin of turbulence ?

Hidden dynamo spins down radiative stars, L. Petitdemange, F. Marcotte, C. Gissinger, Science, editorial revision (2022)





- Subcritical transition to turbulence triggered by a magnetic dynamo
- AGAIN : Ultimate regime for the AM transport in radiative stars (despite a very different source for turbulence)

⇒See poster in session S02 (PNPS)

THANK YOU

1. *Turbulence in electromagnetically-driven Keplerian flows,*
M. Vernet, M. Pereira, S. Fauve, C. Gissinger, ***Journal of Fluid Mechanics, 924, A29*** *(2021)*

2. *Angular momentum transport by Keplerian turbulence in liquid metals*
M. Vernet, S. Fauve, C. Gissinger, accepted in ***Physical Review Letter*** *(2022)*

3. *Hidden dynamo spins down radiative stars*
L. Petitdemange, F. Marcotte, C. Gissinger, ***Science****, editorial revision (2022)*